## Functions

1 a Express $x^{2}-8 x+18$ in the form $(x+a)^{2}+b$.
b Find the distance of the vertex of the curve $y=x^{2}-8 x+18$ from the origin, giving your answer in the form $k \sqrt{5}$.
c Describe two transformations that would map the graph of $y=x^{2}$ onto the graph of $y=x^{2}-8 x+18$.

2


The diagram shows the graph of $y=\mathrm{f}(x)$ which meets the coordinate axes at the points $(-2,0),(0,-4)$ and $(2,0)$.
Showing the coordinates of any points of intersection with the coordinate axes, sketch on separate diagrams the graphs of
a $y=\frac{1}{2}|\mathrm{f}(x)|$,
b $y=4+\mathrm{f}(x+2)$.
3 Sketch the curve with equation $y=2-2 \sin x$ for $x$ in the interval $0 \leq x \leq 2 \pi$.
Label on your sketch the coordinates of any maximum or minimum points and any points where the curve meets the coordinate axes.

$$
\mathrm{f}(x) \equiv|2 x+5|, x \in \mathbb{R}
$$

a Sketch the graph $y=\mathrm{f}(x)$, showing the coordinates of any points where the graph meets the coordinate axes.
b Evaluate $\mathrm{ff}(-4)$.

$$
\mathrm{g}(x) \equiv \mathrm{f}(x+k), x \in \mathbb{R}
$$

c State the value of the constant $k$ for which $g(x)$ is symmetrical about the $y$-axis.
5


The diagram shows the curve $y=a+b \sin x, \quad 0 \leq x \leq 360^{\circ}$.
The curve meets the $y$-axis at the point $A(0,2)$ and has a maximum at the point $B\left(90^{\circ}, 7\right)$.
a Find the values of the constants $a$ and $b$.
b Find the $x$-coordinates of the points $C$ and $D$, where the curve crosses the $x$-axis.

6 a Sketch the curve $y=3 \cos 2 x^{\circ}$ for $x$ in the interval $0 \leq x \leq 360$.
b Write down the coordinates of the points where the curve intersects the $x$-axis.
c Write down the coordinates of the turning points of the curve.
7


The diagram shows the curve with equation $y=\mathrm{f}(x)$ which has two stationary points with coordinates $(4,4)$ and $(7,0)$.
Showing the coordinates of any stationary points, sketch on separate diagrams the curves
a $y=1+2 \mathrm{f}(x)$,
b $y=\mathrm{f}(-3 x)$.
8 a Sketch the curve $y=\frac{1}{2}+\sin 3 x$ for $x$ in the interval $0 \leq x \leq 180^{\circ}$.
b Write down the coordinates of the turning points of the curve.
c Find the $x$-coordinates of the points where the curve crosses the $x$-axis.
9 The function f is defined by

$$
\mathrm{f}: x \rightarrow x^{\frac{1}{2}}-2, x \in \mathbb{R}, x \geq 0
$$

Showing the coordinates of any points where each graph meets the coordinate axes, sketch on separate diagrams the graphs of
a $y=\mathrm{f}(x)$,
b $y=2+|\mathrm{f}(x)|$,
c $y=3 \mathrm{f}(x+1)$.
10 Sketch the curve $y=4 \sin \left(x+\frac{\pi}{3}\right)$ for $x$ in the interval $0 \leq x \leq 2 \pi$.
Label on your sketch
i the value of $x$ at each point where the curve intersects the $x$-axis,
ii the coordinates of the maximum and minimum points of the curve.

11

$$
\mathrm{f}(x) \equiv \frac{3 x-5}{x-2}, x \in \mathbb{R}, x \neq 2 .
$$

a Find $\mathrm{f}^{-1}(x)$ and state its domain.
b Hence, or otherwise, solve the equation $\mathrm{f}(x)=4$.
c Find the values of $a$ and $b$ such that

$$
\mathrm{f}(x)=a+\frac{b}{x-2} .
$$

d Hence, describe two transformations that map the graph of $y=\frac{1}{x}$ onto the graph of $y=\mathrm{f}(x)$.

